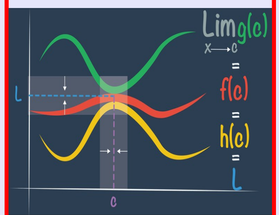
Calculus I

Lecture 24



Feb 19-8:47 AM

Prove $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{[g(x)]^2}$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

LCD: $g(x+h)g(x)$

$$= \lim_{h\to 0} \frac{\cancel{g(x+h)}\,g(x)\cdot\frac{f(x+h)}{\cancel{g(x+h)}} - g(x+h)\,\cancel{g(x)}\cdot\frac{f(x)}{\cancel{g(x)}}}{g(x+h)\,g(x)\cdot h}$$

$$= \lim_{h\to 0} \frac{\overbrace{f(x+h)\,g(x) - f(x)\,g(x)} + \overbrace{f(x)g(x) - g(x+h)f(x)}}{h\,g(x+h)\,g(x)}$$

$$= \lim_{h\to 0} \frac{g(x)\left[f(x+h) - f(x)\right] - f(x)\left[g(x+h) - g(x)\right]}{h\;g(x+h)\,g(x)}$$

$$= \lim_{h\to 0} \frac{g(x)\left[f(x+h) - f(x)\right]}{g(x+h)\,g(x)\;h} - \lim_{h\to 0} \frac{f(x)\left[g(x+h) - g(x)\right]}{g(x+h)\,g(x)\;h}$$

$$= \frac{g(x)}{g(x)\,g(x)}\cdot f'(x) - \frac{f(x)}{g(x)\cdot g(x)}\cdot g'(x)$$

$$= \frac{f'(x)\,g(x) - f(x)\,g'(x)}{g(x)\,g(x)} = \boxed{\frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}}$$

Oct 8-8:32 AM

Find eqn of the tan. line to the graph of $f(x)=\dfrac{2x-1}{x-1}$ at $x=2$.

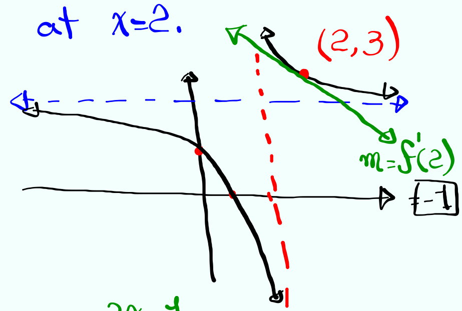
Domain $x \neq 1$

V.A. $x=1$

H.A. $\lim_{x\to\infty} f(x) = 2$

Y-Int $(0,1)$

x-Int $(\frac{1}{2},0)$



$$f(x)=\frac{2x-1}{x-1}$$

$$f'(x)=\frac{2(x-1)-(2x-1)\cdot 1}{(x-1)^2}$$

$$=\frac{\cancel{2x}-2-\cancel{2x}+1}{(x-1)^2}=\frac{-1}{(x-1)^2}$$

$$m=\frac{-1}{(2-1)^2}=\frac{-1}{1}=-1$$

$y-y_1=m(x-x_1)$

$y-3=-1(x-2)$

$\boxed{y=-x+5}$

Oct 9-7:39 AM

Find all points where $f(x)=x^3-3x^2+4$ has horizontal tan. line.

$(\, , \,)$

$m=0$

$f'(x)=0$

$f'(x)=3x^2-6x+0$

$f'(x)=3x^2-6x$

$3x^2-6x=0$

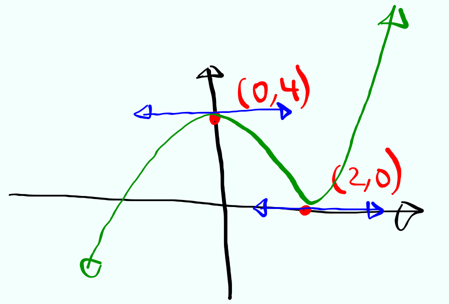
$3x(x-2)=0$

$x=0$ $\quad$ $x=2$

$(0,f(0))$
$=(0,4)$

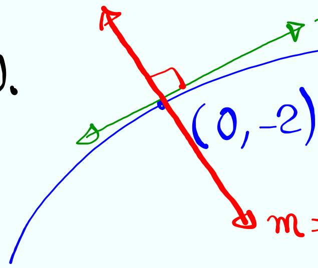
$(2,f(2))=(2,0)$

Guess



Oct 9-7:48 AM

Find eqn of the normal line to the graph of $f(x)=(x^2-3x+1)(x^2+2x-2)$ at $x=0$.



$m=f'(0)=8$

$m=\dfrac{-1}{f'(0)}=\dfrac{-1}{8}$

$y-(-2)=\frac{-1}{8}(x-0)$

$\boxed{y=\frac{-1}{8}x-2}$

$f(x)=(\underbrace{x^2-3x+1})(x^2+2x-2)$

$f'(x)=[2x-3](x^2+2x-2)+(x^2-3x+1)\cdot[2x+2]$

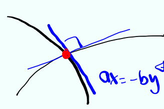
$f'(0)=(-3)\cdot(-2)+1\cdot 2=8$

Oct 9-7:54 AM

Orthogonal curves

Product of derivatives at intersection points is -1.

Tan. lines at their intersection points are perpendicular to each other.



$ax=-by$

show $x^2+y^2=r^2$ and $ax+by=0$ are orthogonal curves.

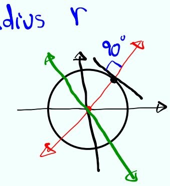
$x^2+y^2=r^2$

Circle, Center at $(0,0)$, radius r

$ax+by=0 \rightarrow y=\frac{-a}{b}x$

line slope $\frac{-a}{b}$

Y-Int $(0,0)$



$ax+by=0$

$\frac{d}{dx}[ax+by]=\frac{d}{dx}[0]$

$a+b\frac{dy}{dx}=0 \qquad a+by'=0 \rightarrow y'=\frac{-a}{b}$

$\frac{d}{dx}[x^2+y^2]=\frac{d}{dx}[r^2] \rightarrow y'=\frac{-2x}{2y}=\frac{-x}{y}$

$2x+2yy'=0$

Implicit Diff.

$\frac{-a}{b}\cdot\frac{-x}{y}$

$=\frac{ax}{by}=\frac{-by}{by}=\boxed{-1}$

Oct 9-8:03 AM

$$\frac{d}{dx}\left[f(x)\right] = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}\left[f(x)\right]\Big|_{x=a} = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$\frac{d}{dx}\left[c\right] = 0 \qquad \frac{d}{dx}\left[c\, f(x)\right] = c\,\frac{d}{dx}\left[f(x)\right]$$

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$$

$$\frac{d}{dx}\left[f(x) \cdot g(x)\right] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left[g(x)\right]^2}$$

Oct 9-8:16 AM

$$\frac{d}{dx}\left[\sin x\right] = \cos x \qquad \frac{d}{dx}\left[\cos x\right] = -\sin x$$

$$\frac{d}{dx}\left[\tan x\right] = \sec^2 x \qquad \frac{d}{dx}\left[\cot x\right] = -\csc^2 x$$

$$\frac{d}{dx}\left[\sec x\right] = \sec x \tan x \qquad \frac{d}{dx}\left[\csc x\right] = -\csc x \cot x$$

Oct 9-8:21 AM

Find $f'(x)$ for $f(x) = \frac{1-\sec x}{\tan x}$

$$f'(x) = \frac{\frac{d}{dx}[1-\sec x]\cdot\tan x - (1-\sec x)\cdot\frac{d}{dx}[\tan x]}{[\tan x]^2}$$

$$= \frac{-\sec x\tan x\tan x - (1-\sec x)\cdot\sec^2 x}{\tan^2 x}$$

$$= \frac{-\sec x\tan^2 x - (1-\sec x)\sec^2 x}{\tan^2 x}$$

Recall
$1+\tan^2 x = \sec^2 x$

$$= \frac{-\sec x\tan^2 x - (1-\sec x)(1+\tan^2 x)}{\tan^2 x}$$

$$= \frac{\cancel{-\sec x\tan^2 x} - 1 - \tan^2 x + \sec x + \cancel{\sec x\tan^2 x}}{\tan^2 x}$$

$$= \frac{\sec x - (1+\tan^2 x)}{\tan^2 x} = \frac{\sec x - \sec^2 x}{\tan^2 x}$$

$\sec^2 x - 1$

$$= \frac{-\sec x(\cancel{\sec x - 1})}{(\sec x+1)\cancel{(\sec x - 1)}} = \boxed{\frac{-\sec x}{\sec x + 1}}$$

Oct 9-8:23 AM

Google or Look up

$\boxed{\text{Chain Rule}}$

Oct 9-8:34 AM